

CALCULATION OF THE HEATING OF A METAL BY A MOVING SOURCE,
WITH CONSIDERATION OF SURFACE OXIDATION

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UDC 536.2

We have observed that in problems dealing with the heating of a plate and a half-space by a moving source, with consideration of surface oxidation, the quasisteady solution is multivalued.

The problem of heating metals with concentrated sources of energy is presently of considerable interest. The temperature fields that arise within a plate and a half-space when subjected to heating with nonmoving and moving surface heat sources, in the case of constant values for the thermophysical quantities, have been studied in detail in [1, 2]. The thermophysical coefficients in their dependence on temperature have been accounted for in [3, 4], while in [5, 7] we find calculations of the processes of metal heating by a nonmoving source in an oxidizing atmosphere. The problem of heating a semibounded body with a moving spot source has been examined in [8] with consideration given to the oxidation of the surface, and it was demonstrated that a change in the coefficient of absorption with a reduction in velocity is discontinuous in nature. However, no detailed calculations in the region of the "discontinuity" were undertaken. In the present paper we examine the problem of using a moving source to heat a plate and a half-space, while making provision for changes in absorptivity due to oxidation. A detailed study has been undertaken here into the behavior of the solution and we have observed the existence of regions of parameters in which it is multivalued.

Let us examine heating in an oxidizing atmosphere of a semibounded metal body by means of a spot source of power Q , moving at velocity v . An oxidation reaction takes place at the surface of the metal and an oxidation layer is produced. Let us examine the quasisteady problem, neglecting the growth of the oxide film at temperatures close to the initial temperature T_0 . In this case, the distribution of the temperature through the metal, without consideration of exchange through the thickness of the oxide, nor without consideration given to the phase transitions and to the exchange of heat with the ambient medium, is determined from the following formula [1]

$$T = \frac{A_s Q}{2\pi\lambda R} \exp\left(\frac{vx}{2a} - \frac{vR}{2a}\right) + T_0. \quad (1)$$

The origin of the Cartesian x, y coordinate system is connected to the moving source: the direction of the x axis is opposite to that of the vector v .

Generally speaking, the absorption factor is a nonmonotonic function of the oxide thickness ξ [9]. Prior to the first maximum, following [7], we will take the quadratic approximation $A(\xi)$; with larger ξ we will assume that the coefficient of absorption is constant:

$$A(\xi) = \min \{A_0(1 + b\xi^2), A_0(1 + b\xi_m^2)\}, \quad (2)$$

with $A_s = A(\xi_s)$, $\xi_s = \xi|_{x=y=0}$.

The rate of oxide film growth is initially determined by the parabolic oxidation law [10], subsequent to which the gas-phase limitation regime sets in, and here the rate of oxide growth will be assumed to be constant [11]. Thus, for the points of the x axis we have:

$$\frac{d\xi}{dx} = \min \left\{ \frac{D}{\xi v} \exp\left[-\frac{T_d}{T(x)}\right]; \frac{v_t}{v} \right\}. \quad (3)$$

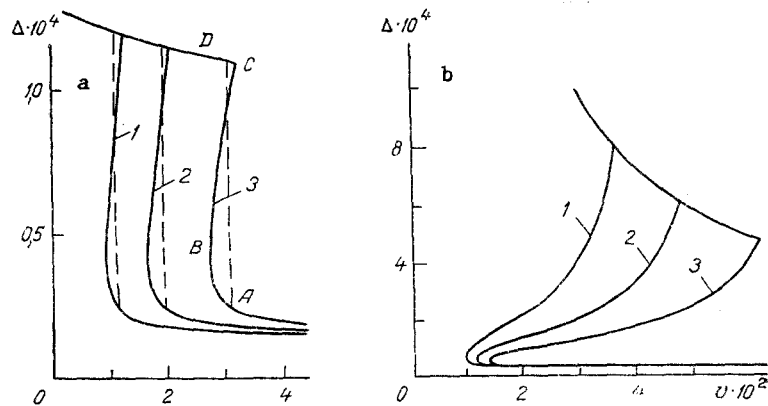


Fig. 1

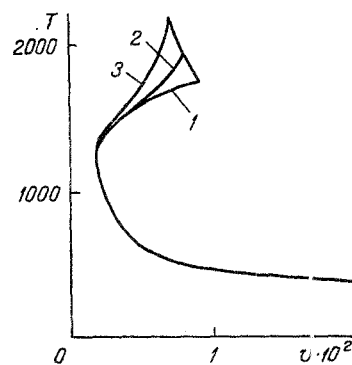


Fig. 2

Fig. 1. The half-width of the surface area heated higher than 1000 K as a function of the velocity of source motion: a) half-space; b) plate; 1) $v_0 = 3.4 \cdot 10^{-4}$; 2) $6.1 \cdot 10^{-4}$; 3) $10.6 \cdot 10^{-4}$ m/sec; dashed curves represent the results from [8]. Δ , m; v , m/sec.

Fig. 2. The temperature behind the source as a function of the velocity of strip-source motion over the plate, $v_0 = 3.4 \cdot 10^{-4}$ m/sec: 1) $d = 0$; 2) 0.4; 3) 1 mm. T , K.

In accordance with (1)

$$T(x) = \frac{A_s Q}{2\pi\lambda|x|} \exp\left(\frac{vx}{a}\right) + T_0 \text{ when } x < 0.$$

Integrating (3) from $-\infty$ to 0 (with $T < 400$ K $d\xi/dx$ assumed to be equal to zero) with the initial condition $\xi = \xi_0$ when $x = -\infty$ and using the condition $\xi(0) = \xi_S$, we obtain an equation for the determination of ξ_S (and, consequently, for the determination of A_S). All subsequent calculations were performed for values of $\lambda = 20$ W/(m·deg), $a = 10^{-5}$ m/sec, $D = 3.3 \cdot 10^{-2}$ m²/sec, $T_d = 3.3 \cdot 10^4$ K, $b = 2 \cdot 10^{12}$ m⁻², $A_0 = 0.1$, $\xi_M = 2$ μ m, corresponding to Ti; $T_0 = 295$ K, $\xi_0 = 0.2$ μ m.

Figure 1a shows the half-width of the surface area heated higher than 1000 K as a function of the velocity of source motion in the case of $Q = 12.6$ W for various values of v_0 . As we can see from the figure, in all three cases, the quasisteady solution in the region of the "discontinuities" in [8] is not uniquely defined. Three values of Δ (and correspondingly, of A_S) correspond to a single value for the displacement velocity of the source in some range of velocities. This leads to a situation in which, depending on the direction of the change in velocity, the discontinuity in the quasisteady solution comes about for a variety of solution values. Indeed, if the velocity of source motion is gradually reduced from the value corresponding to the point A (Fig. 1a) to a value corresponding to the point B, the half-width Δ will increase smoothly in accordance with the curve AB. However, with a further reduction in velocity Δ will increase discontinuously to values corresponding to the point D. If the velocity is then increased, the reverse discontinuity to the curve AB will now occur at point C. Let us note that the discontinuous change in the width of the heated zone will actually occur at times on the order of the time required to establish the quasisteady temperature distribution within the metal. The BC segment in which $d\Delta/dv > 0$, apparently, is not stable and can not be achieved in actual practice. Indeed, with a slight increase in the velocity of source motion, the temperature of the metal surface becomes an increasingly pronounced diminishing function of the distance from the source in the direction of its motion. Moreover, as a consequence of the increase in the velocity of motion, the oxidation time for the surface point is reduced to that instant of time at which it passes through its source. The thickness ξ_S of the oxide layer, the absorptivity A_S and, consequently, Δ are therefore reduced. This leads to a descent from the segment BC.

Let us note that the physical nature of nonunique definition is associated, at least for the curves CD and AB, with the fact that the value for the power of the source is one and the same, and its effective values diverge significantly because the corresponding coefficients of absorption differ several times over. Indeed, in the AB segment the absorptivity is small and the temperature ahead of the source is therefore inadequate for an intensive growth of the oxide. This leads in turn to retention of limited absorptivity. In the CD segment the absorptivity is high and the temperature ahead of the source is sufficient for

segment the absorptivity is high and the temperature ahead of the source is sufficient for intensive growth of the oxide and absorption consequently remains high.

Results from analogous calculations for a thin plate heated by a moving spot source with a linear power of $Q_1 = 2 \cdot 10^5$ Q/m can be found in Fig. 1b. The temperature distribution in this case was determined from the following formula [1]:

$$T = T_0 + \frac{A_s Q_1}{2\pi\lambda} \exp\left(\frac{xv}{2a}\right) K_0\left(\frac{rv}{2a}\right),$$

where K_0 is a Bessel function of imaginary argument.

As we can see from Fig. 1, in the cases under consideration here, the region of nondefiniteness in the problem pertaining to the heating of a plate is considerably broader than in the problem encountered in the heating of a half-space.

Let us also examine the one-dimensional problem of the heating of a plate by a linear source with the linear density $Q_2 = 3 \cdot 10^7$ W/m². The temperature distribution in this case is described by formula (1)

$$T = T_0 + \frac{A_s Q_2 a}{\lambda v} \exp\left(\frac{vx}{2a} - \frac{v|x|}{2a}\right).$$

Results from corresponding calculations are shown in Fig. 2 (curve 1) in the form of a relationship between the temperature T_s at the center of the source and the velocity. As we can see from the figure, the effect of nondefiniteness in the quasisteady solution is present in this case as well.

For purposes of examining the effect that the finiteness of the source dimensions has on the nature of the quasisteady solution, we carried out appropriate calculations for strip sources with a constant intensity and the same total linear power. Figure 2 shows the results of such calculations for sources of width $d = 0.4$ and 1 mm. As we can see, the nondefiniteness of the relationship between the temperature T_s behind the source and v is retained in this case, although it becomes less clearly defined as the width of the source is increased.

Thus, consideration of the metal surface oxidation in the case of heating with a shifting concentrated source of energy significantly affect the heating pattern. The quasisteady solution is not uniquely defined and the position of the boundaries of the nonunique region depends strongly on the magnitude of the rate of oxide growth in the gas-phase limitation regime.

NOTATION

T , surface temperature; x, y, z , Cartesian coordinates; $R = \sqrt{x^2 + y^2 + z^2}$, $r = \sqrt{x^2 + y^2}$; λ, a, A , coefficients of thermal conductivity, thermal diffusivity, and absorption; v , velocity of source motion; Q , source power; ξ , oxide thickness; D , parabolic oxidation law constant; T_d , activation energy of thermodynamic reaction; v_d , rate of oxide growth in gas-phase limitation regime; Δ , half-width of surface area heated above 1000 K; $Q_{1,2}$, linear values of source power. Subscripts: 0, initial value; s, value at coordinate origin.

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